APPENDIX II

DERIVATIONS OF FORMULAS FOR ASSEMBLY INTERFERENCES

The interferences Δ_n calculated in the text are the interferences required on the component parts as manufactured. However, the manufactured interference is not equal to the interference as assembled. The multiring container is taken as an example. It is assumed the rings are shrink-fit assembled one-by-one from the inside. The outer rings expand as they are shrunk on and the assembly interference for the next ring to be fitted is increased beyond the manufactured interference. The assembly interference between cylinders n and n + 1 is denoted by δ_n. It has dimensions of inches.

For assembly of cylinder n + 1 onto the other cylinders, δ_n is expressed as

$$\frac{\delta_n}{r_n} = \frac{\Delta_n}{r_n} + \frac{u_n'(r_n)}{r_n} \tag{110}$$

where

$u_n'(r_n)$ = radial displacement at r_n of cylinder n due to residual pressure q_{n-1}' at r_{n-1}.

q_{n-1}' = residual pressure at r_{n-1} due to assembly of cylinder n of wall ratio k_n onto a compound cylinder of wall ratio $k_1 k_2 \ldots k_{n-1}$ with an interference δ_{n-1}.

q_{n-1}' is calculated as follows:

$$\frac{\delta_{n-1}}{r_{n-1}} = \frac{u_n(r_{n-1}) - u_{n-1}(r_{n-1})}{r_{n-1}}$$

Substitution for u_n and u_{n-1} from Equation (14a) gives

$$\frac{\delta_{n-1}}{r_{n-1}} = \frac{1}{E_n(k_n^2-1)}\left[(1-v)\, q_{n-1}' + (1+v)\, q_{n-1}'\, k_n^2\right]$$

$$- \frac{1}{E_{n-1}(k_{n-1}^2\, k_{n-2}^2 \ldots k_1^2 - 1)}\left[-(1-v)\, q_{n-1}'\, k_{n-1}^2\, k_{n-2}^2 \ldots k_1^2 - (1+v)\, q_{n-1}'\right]$$

$$= \frac{q_{n-1}'}{E}\left[\frac{k_n^2 + 1}{k_n^2 - 1} + \frac{k_{n-1}^2\, k_{n-2}^2 \ldots k_1^2 + 1}{k_{n-1}^2\, k_{n-2}^2 \ldots k_1^2 - 1}\right]$$

where $E_n = E_{n-1} = E$ is assumed.

$$\text{Hence, } q'_{n-1} = E\left(\frac{\delta_{n-1}}{r_{n-1}}\right)\frac{\left(k_n^2-1\right)\left(k_{n-1}^2\, k_{n-2}^2 \cdots k_1^2-1\right)}{2\left(k_n^2\, k_{n-1}^2\, k_{n-2}^2 \cdots k_1^2 - 1\right)} \tag{111}$$

Since

$$\frac{u'_n(r_n)}{r_n} = \frac{2q'_{n-1}}{E\left(k_n^2-1\right)} \tag{112}$$

Substitution of (111) and (112) into (110) gives

$$\frac{\delta_n}{r_n} = \frac{\Delta_n}{r_n} + \frac{\delta_{n-1}}{r_{n-1}}\frac{\left(k_{n-1}^2\, k_{n-2}^2 \cdots k_1^2 - 1\right)}{\left(k_n^2\, k_{n-1}^2\, k_{n-2}^2 \cdots k_1^2 - 1\right)} \tag{113}$$

Now the $\frac{\delta_n}{r_n}$ can be calculated in sequence; i.e.,

$$\frac{\delta_1}{r_1} = \frac{\Delta_1}{r_1}$$

$$\frac{\delta_2}{r_2} = \frac{\Delta_2}{r_2} + \frac{\delta_1}{r_1}\frac{\left(k_1^2 - 1\right)}{\left(k_1^2\, k_2^2 - 1\right)} \quad \text{, etc.}$$

Equation (113) applies if the rings are assembled from the inside out. If the rings are assembled one by one from the outside in, then the assembly interference for assembly of cylinder n-1 into the other cylinders is

$$\frac{\delta_n}{r_n} = \frac{\Delta_n}{r_n} + \frac{\delta_{n+1}}{r_{n+1}}\frac{k_n^2 + 1 \quad \left(k_{n+1}^2\, k_{n+2}^2 \cdots k_N^2 - 1\right)}{\left(k_{n+1}^2 k_{n+2}^2 \cdots k_N^2 - 1\right)} \tag{114}$$

Equation (114) was found by an analogous procedure to that used in deriving (113).

The method used to determine assembly interferences δ_n for the multiring container can also be used to determine assembly interferences for the other container designs. It is important to determine assembly interferences because they are larger than the manufactured interferences and excessive interference requirements may make a design impracticable.